The Perturbative Pomeron and the Odderon: Where can we find them? *

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Abstract

QCD predicts the existence of the perturbative Pomeron and of the Odderon. But both of them appear to be rather difficult to observe experimentally. We describe the experimental status of these two objects, discuss possible reasons for their elusive behavior, and point out promising search strategies.

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1 Introduction

One of the most interesting problems in QCD is to understand the high energy limit of hadronic scattering processes. Already before the advent of QCD this problem had been widely studied in the framework of Regge theory, for a recent review see [1]. Based on the principles of analyticity, unitarity and Lorentz invariance of the scattering amplitude Regge theory gives an extremely successful phenomenological description of strong interactions in the Regge limit, i.e. in the limit of large center-of-mass energy $\sqrt{s}$ and relatively small momentum transfer $\sqrt{-t}$, the latter being chosen to be of the order of a hadronic mass scale. It is convenient in Regge theory to change from the squared energy s to its conjugate variable, the complex angular momentum ω, and via an integral transformation the scattering amplitude is obtained as a function of ω. Regge theory then relates the high energy behavior of hadronic scattering processes to the singularities of the scattering amplitude in the complex angular momentum plane, the Regge poles and Regge cuts. The leading contribution in the high energy limit is given by the rightmost singularity in the ω-plane, the Pomeron. It can be interpreted as a t-channel exchange carrying vacuum quantum numbers between the scattering particles. In the framework of Regge theory the positions of the Regge singularities (together with their couplings to the scattering particles) are universal parameters which have to be determined from experimental data. Once known they can be used to predict cross sections for other processes. Regge theory is very successful in describing the wealth of available data on hadronic cross sections, including total cross sections, structure functions at small values of Bjorken-x as well as diffractive processes. The high energy behavior of total hadronic cross sections for example is very well described by a Pomeron with intercept 1.09 [2], leading to a slowly rising cross section $\sigma \sim s^{0.09}$. Obviously, one would like to derive Regge theory from QCD. But this is a difficult problem, mainly because scattering processes in the Regge limit are in general dominated by small momentum scales. A derivation of the Regge singularities from first principles would hence require a good understanding of nonperturbative QCD which we clearly do not have at present.

In this talk I will address two issues which can help in making progress towards an understanding of Regge theory in terms of QCD. The first is the perturbative approach to the Pomeron which is a first step in the direction of deriving Regge singularities from QCD. The second is the Odderon, the partner of the Pomeron carrying negative charge parity quantum number. Although its existence is expected on the basis of our picture of high energy scattering in QCD, the Odderon has so far escaped unambiguous experimental detection. I will discuss mainly phenomenological aspects of the perturbative Pomeron and of the Odderon, in particular the presently available evidence for their existence.

2 The perturbative Pomeron

The perturbative approach to high energy scattering is based on the resummation of large logarithms of the center–of–mass energy $\sqrt{s}$. The applicability of perturbation theory clearly requires that the value of the strong coupling constant can be assumed to be small in the scattering process under consideration. This means that the scattering

process involves at least one large momentum scale. (Note that in a quantum field theory like QCD the running coupling constant always depends on the momentum of the particles, but not on the center–of–mass energy.) As is the case for any approximation scheme in physics one has to determine its range of applicability. As we will discuss below it turns out that in the case of the Pomeron there are additional effects which limit the applicability of perturbation theory. As a consequence the perturbative approach to the Pomeron can be used only in a rather limited number of scattering processes. Nevertheless, it is of enormous theoretical importance because it is, at least for the time being, the only rigorous way to understand the origin of Regge behavior of some scattering processes in terms of QCD.

In the leading logarithmic approximation (LLA) one collects all diagrams of the perturbative series in which factors of the strong coupling constant α_s are accompanied by a logarithm of the energy $\sqrt{s}$. At high energies these logarithms can compensate the smallness of the strong coupling, and hence the LLA is characterized by

$$\alpha_s \ll 1 \,, \qquad \alpha_s \log s \sim 1 \,. \tag{1}$$

The resummation of all perturbative terms of the form $(\alpha_s \log s)^n$ in a two–particle scattering process was performed in [3, 4], and the exchange in the t-channel of the resulting amplitude is known as the BFKL (Balitsky–Fadin–Kuraev–Lipatov) Pomeron. The BFKL equation describing that object is an evolution equation in energy. Equivalently, it can be interpreted as an evolution in rapidity along the t-channel or in longitudinal momentum of the real partons produced in the scattering process. The BFKL equation does however not include the effects of an evolution in the virtuality of the particles along the t-channel exchange. It is therefore strictly speaking only applicable to scattering processes which are completely dominated by only one hard momentum scale. The best processes for the study of the perturbative Pomeron are therefore scattering processes of two small color dipoles, and the size of the dipoles determines the hard scale necessary for the perturbative treatment. Theoretically, the best possible process would be the scattering of two heavy onia, which is however not accessible experimentally. Realistic processes which are close to this ideal situation are[1] Mueller–Navelet jets (i. e. forward jets) in proton–(anti)proton scattering [5], the production of hard forward jets in deep inelastic lepton–nucleon scattering[6], as well as the scattering of two virtual photons of the same or at least similar virtuality. In the latter case one can consider either quasi–diffractive processes like for example $\gamma^*\gamma^* \to J/\psi J/\psi$ [7], or the total hadronic cross section in $\gamma^*\gamma^*$ collisions [8, 9, 10, 11].

Before considering a concrete scattering process let us first discuss some basic properties of the BFKL Pomeron. The diagrams resummed in the LLA are of the form shown in figure 1. The figure shows a typical diagram contributing to the amplitude. A gluon is exchanged in the t-channel from which a number of real gluons can be emitted in the s-channel. In the LLA also virtual corrections are included which lead to the so–called reggeization of the t-channel gluon. In the process of reggeization the t-channel gluon becomes a more complicated object (it becomes a collective excitation of the gluon field rather than an elementary gluon), but for the present talk it will be sufficient to think of a gluon exchange in the t-channel. When the amplitude is squared

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[1]Here I give only early references for the different processes. For further relevant references see papers referring to these.
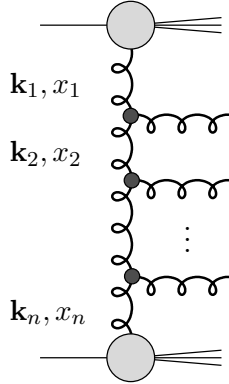
Figure 1: Real gluon emissions from the t-channel gluon

in order to obtain the cross section one finds diagrams which exhibit the characteristic ladder structure of the perturbative Pomeron. The gluons along the ladder are strongly ordered in rapidity, or alternatively in longitudinal momentum fraction x_i. Only these strongly ordered configurations give rise to a logarithm of the energy $\sqrt{s}$ for each factor of the coupling constant α_s. The transverse momenta $\mathbf{k}_i^2$ of the emitted gluons, on the other hand, are not ordered. This has to be contrasted with the situation in DGLAP evolution [12, 13, 14] where similar diagrams occur, but with the transverse momenta strongly ordered along the ladder. The BFKL equation is usually written as an integral equation in which the integral kernel represents a rung of the ladder, i. e. the emission of a real gluon. Solving the BFKL equation one finds for the energy dependence of the cross section in the LLA a powerlike growth, $\sigma \sim s^{\omega_{\mathrm{BFKL}}}$, with the BFKL exponent given by

$$\omega_{\mathrm{BFKL}} = \frac{\alpha_s N_c}{\pi} \, 4 \log 2 \, . \tag{2}$$

It is of the order of 0.5 when a typical value of 0.2 is chosen for the strong coupling constant α_s. The so–called Pomeron intercept is hence given by $\alpha_{\mathbf{P}} = 1 + \omega_{\mathrm{BFKL}}$. The LLA does not include the running of the coupling constant α_s. Strictly speaking one hence has to use a fixed α_s in this approximation, although it is widely believed that replacing a fixed value by the running coupling is an improvement. Note that the energy dependence of the cross section in the LLA depends exponentially (read: very strongly) on the coupling constant α_s. Given the above formula this is a rather trivial observation. Nevertheless, it constitutes a rather important limitation for making accurate predictions within this approximation scheme because the determination of the appropriate value of α_s, or equivalently of the relevant momentum scale, is often rather difficult in practice. Due to that any prediction made in the framework of the LLA has an unavoidable uncertainty which, although its origin is trivial, can be quite large. The only way to avoid this problem at least partially is to go to the next–to–leading logarithmic approximation (NLLA) in which running coupling effects naturally occur. We will discuss that approximation further below.

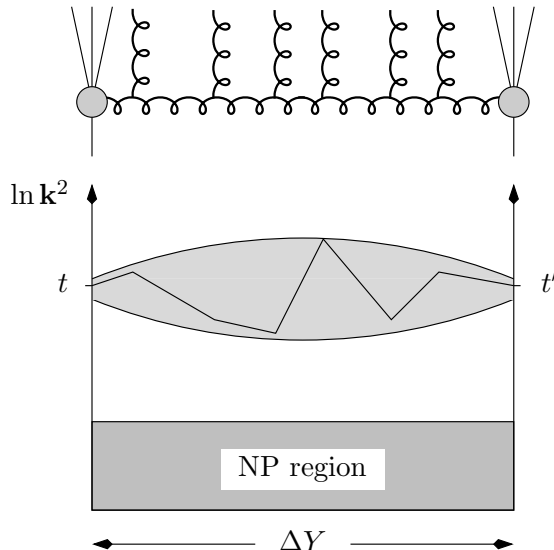An advantage of the BFKL equation is that it can be solved analytically. The

Figure 2: Diffusion of transverse momenta in the BFKL Pomeron

solution represents ladder diagrams with arbitrarily many rungs. An important point to note here is that in the LLA the gluons emitted from the t-channel gluon can be produced without any cost in energy. Hence energy conservation is violated at the emission vertices. This is not because the authors had not been informed about energy conservation in Nature, but is simply an outcome of the approximation scheme. In the sense of leading logarithms of the energy the effect of the correct kinematics of the vertices is a subleading effect. The formally subleading effect of energy–momentum conservation at the emission vertices has been studied by implementing a so–called consistency constraint in the BFKL equation [15]. In a Monte Carlo study of the correspondingly modified BFKL evolution it was found that the constraint considerably reduces the growth of the cross section with the energy [16]. Physically this means that the production of a real gluon requires some amount of energy, and realistically we should not expect arbitrarily large numbers of gluons to be produced. We will come back to this effect further below.

Another phenomenologically important property of the BFKL Pomeron follows immediately from the fact that the transverse momenta $\mathbf{k}_i^2$ of the gluons are not ordered along the ladder. As a consequence there is nothing that prevents the transverse momenta from becoming arbitrarily small. One in fact finds that the gluon emissions along the ladder lead to a random walk in $\log \mathbf{k}_i^2$. The resulting probability distribution of momenta along the ladder resembles a diffusion process. This is illustrated in figure 2. Here the horizontal axis shows the rapidity interval between the ends of the ladder, $\Delta Y \sim \log(s)$. The vertical axis shows the logarithm of the transverse momentum which is fixed at the ends of the ladder at values $t = t'$ determined by the typical momentum scales of the external particles. The resulting probability distribution is known as the Bartels cigar. The exact shape of the cigar depends on the external momentum scales and on the rapidity interval available for evolution [17, 18]. With increasing energy in the scattering process the momentum distribution becomes wider in the middle. There-
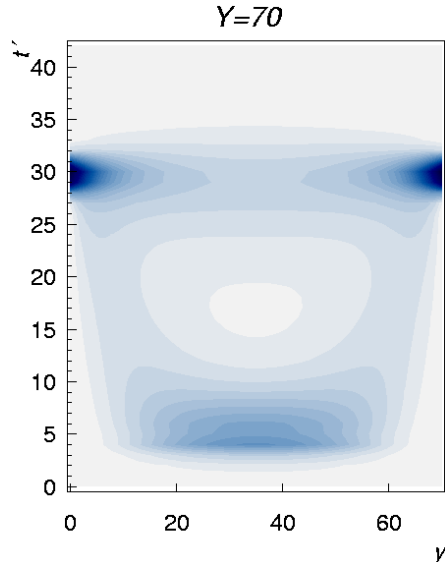
Figure 3: Transition from cigar–type evolution to tunneling–type evolution, figure from [19]

fore some contribution from the nonperturbative region of small momenta cannot be avoided at very high energies even when the external momenta are chosen very large. But in situations in which this contribution is large the whole perturbative description using the BFKL Pomeron is no longer applicable. The situation is particularly severe if the external particles provide small momentum scales, and this already indicates that the BFKL Pomeron cannot be used for the description of structure functions of the nucleon since there one end of the ladder resides completely in the nonperturbative region.

Interestingly, the situation becomes even worse when the coupling constant α_s is assumed to run as a function of the gluon momenta along the ladder [19]. In that case the probability distribution of the transverse gluon momenta is no longer symmetric in the vertical direction. Instead emissions with smaller momenta become more likely as the coupling constant α_s is larger at smaller momenta. Due to that the distribution takes a banana shape rather than the cigar shape. At very large energies, or equivalently large rapidity intervals for evolution, even a tunneling transition takes place. Then the first emission brings the gluon into the infrared region where it stays until the last step of the evolution. A numerical simulation of such a situation is shown in figure 3. In such a situation the perturbative description breaks down completely and the process is determined completely by the soft Pomeron. Note that even for large external momentum scales t this eventually happens as the energy becomes very large. The problem of diffusion of the transverse momenta is an important limitation for the applicability of the BFKL Pomeron. Fortunately, one can find out via numerical simulation whether a given process involves a large contribution from the nonperturbative region or not. In addition, one can choose suitable cuts in a number of scattering processes such that the external momenta are large enough to suppress the diffusion into the infrared. We will now see an example for such a process.
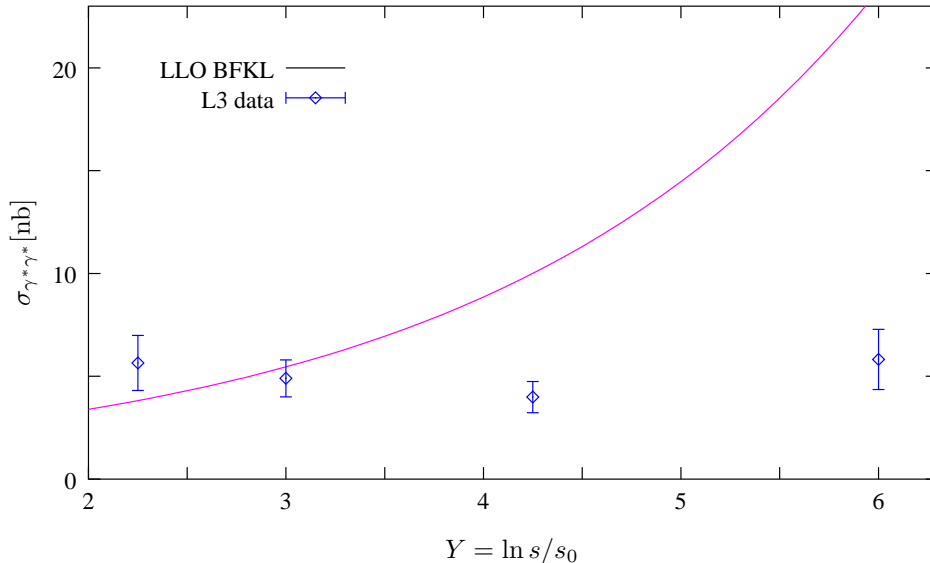
Figure 4: Data for the total hadronic cross section in $\gamma^*\gamma^*$ collisions at LEP2 as measured by the L3 collaboration [22] compared to the LLO BFKL prediction of [20]

Let us now consider the total hadronic cross section of virtual photon–photon scattering as a specific example of a process in which one would expect to see effects of the perturbative Pomeron, namely a rise of the cross section with energy. As already mentioned in the introduction this process is one of the best possible probes of BFKL dynamics at least from a theoretical point of view. This process has been studied in e^+e^- collisions at LEP. Here one selects so–called double–tagged events in which both the scattered electron and positron are detected and the virtualities of the photons emitted from the two can be reconstructed. If the two photon virtualities are chosen large enough and of the same order of magnitude the process is in fact determined by only one hard momentum scale. This process was first studied in [8, 9, 10, 11], later on these studies were refined in various ways, including attempts at including NLLO effects. Here we show results obtained in [20] using the LLA. That calculation in particular includes the effect of the charm quark mass which is quite important for this process. The corresponding theoretical expectations based on the LLA are shown in figure 4. The cross section has been measured by the OPAL and L3 collaborations at LEP [21, 22], and figure 4 shows the L3 data from [22]. There are still quite a few uncertainties in the theoretical expectation, in particular corresponding to the appropriate choice of α_s and of the fixed energy scale s_0. This scale s_0 should be a characteristic scale for the process, here it is chosen as the geometric mean of the two photon virtualities. (Strictly speaking it can only be fixed in a NLLA calculation.) But these uncertainties do by no means affect the obvious conclusion that the data remain far below the BFKL expectation based on the LLA. At least at energies accessible at LEP a strong rise of the cross section is clearly not visible.

This result raises the question whether the data can be explained completely without any resummation of large logarithms of the energy. In order to answer this question the data have been compared with a fixed order calculation in [23]. The calculation
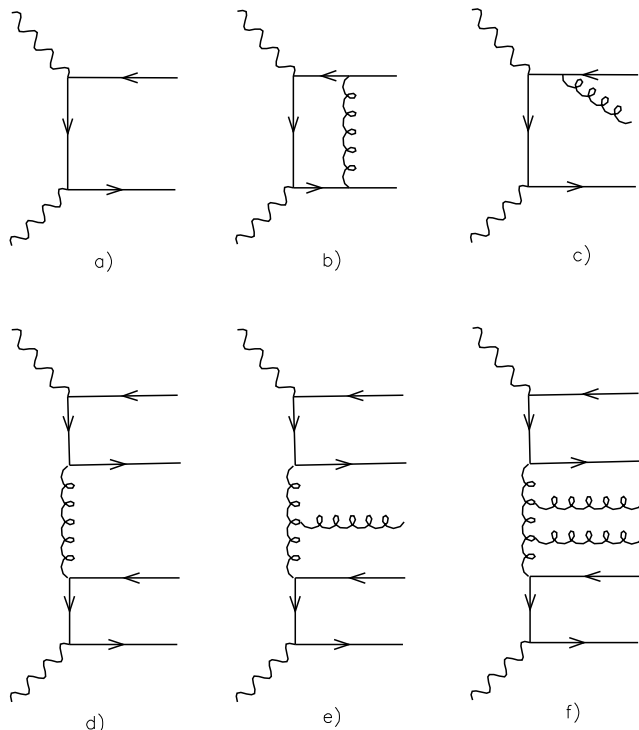
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Figure 5: Typical diagrams for the production of hadrons in $\gamma^*\gamma^*$ collisions, figure from [23]

performed there is done in next–to–leading order (NLO) and hence includes diagrams of the type a)–c) in figure 5. At high energies one expects that diagrams of the type d)–f) become increasingly important since they contain the enhancement due to logarithms of $\sqrt{s}$. The diagrams of type a)–c) on the other hand involve an exchange of a fermion in the t-channel. It is known that these diagrams are necessarily suppressed at high energies by a power of $\sqrt{s}$. At large energies one therefore expects at least the diagram of type d) to become important, which does not have any energy dependence. The actual BFKL type enhancement only starts with the diagram of type e). The result of the NLO calculation is shown in figure 6, here on the level of the e^+e^- cross section. The pure NLO curve is well below the data especially at large energies. If one includes the diagrams of type d) one obtains the solid curve which is only slightly below the data. The calculation in [23] is performed with four massless quark flavors. In reality the mass of the charm quark suppresses the production rate of charm quarks in this process, and according to [20] the calculation with a massless charm quark overestimates the actual cross section by about 15 %. Taking this effect into account one concludes from figure 6 that the data point at the highest energy actually exceeds the expectation of a fixed order calculation for this process. But the enhancement at high energy is by far not as large as predicted by BFKL resummation in LLA. As we will explain further below a consistent calculation of this prediction in next–to–leading logarithmic accuracy is not available at present.

Also in other processes in which the perturbative Pomeron can be looked for the

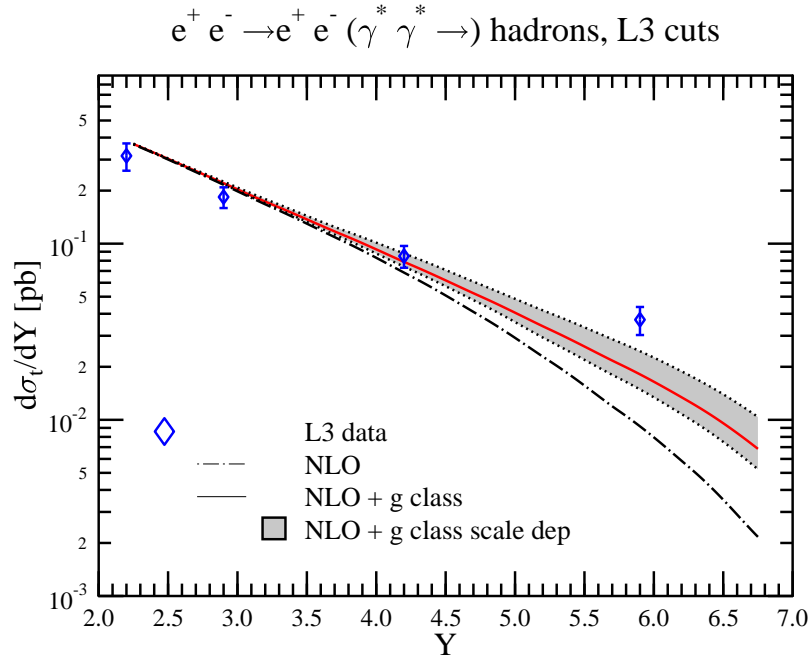$$e^+ e^- \rightarrow e^+ e^- (\gamma^* \gamma^* \rightarrow) \text{ hadrons, L3 cuts}$$

Figure 6: Prediction for the total hadronic cross section for the process $e^+ e^- \rightarrow e^+ e^- X$ in NLO (dot-dashed line), and NLO plus diagrams of type d) in figure 5 (solid line), together with the data from [22], figure from [23]

situation is similar. An unambiguous sign of the BFKL Pomeron has not yet been found. One often finds an enhancement at high energies, but it is usually much smaller than one would have expected in LLA. We are thus led to the conclusion that the LLA is not applicable in the situations that are experimentally accessible at present. In each case there are different problems, but one key problem is common to all these cases. This issue is closely related to the problem of the violation of energy conservation in the LLA that we have already discussed. The analytic solution of the BFKL equation contains diagrams with arbitrarily many gluon rungs. In reality, however, the emission of a gluon into the final state requires a certain amount of energy. Our experience with deep inelastic scattering at HERA tells us that this amount of energy can roughly be estimated to correspond to one unit in rapidity. Events corresponding to the highest available energies at HERA of LEP for example span a rapidity range of about 5 or 6 units. In addition we have to take into account that also the breakup of the incoming particles, for example the transformation of a virtual photon into a quark–antiquark pair, takes up at least one unit in rapidity. In total we should therefore expect that in reality only two or three gluons can be emitted. It is therefore not at all surprising that at these energies a fixed order calculation comes close to the measured cross section.

As already mentioned above the problem of energy conservation at the emission vertices is closely related to next–to–leading logarithmic corrections to the BFKL equation. The derivation of the BFKL equation in NLLA is much more difficult than the LLA version. In an effort that lasted for almost ten years that problem of including terms
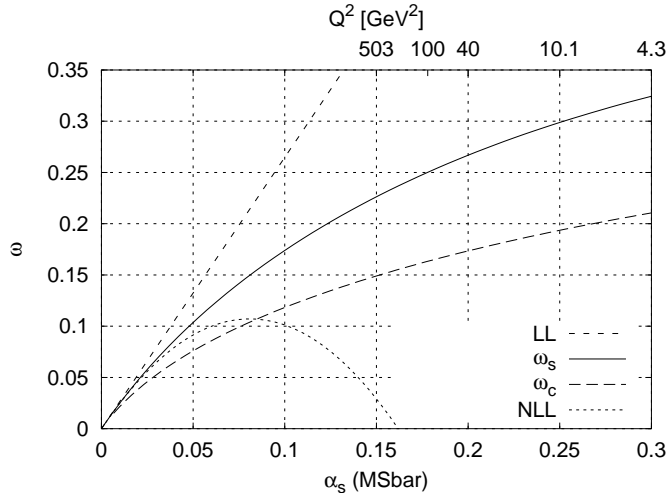
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Figure 7: BFKL exponent as a function of the coupling constant α_s using a renormalization group improvement of the NLLO BFKL equation, figure from [28]

of the order $\alpha_s(\alpha_s \log s)^n$ has been solved, see [24, 25] and references therein. The corrections were found to be rather large, giving for the characteristic exponent ω_{BFKL} of the energy dependence

$$\omega_{\mathrm{BFKL}} \simeq 2.65 \, \alpha_s (1 - 6.18 \, \alpha_s) \,, \tag{3}$$

where the first term corresponds to the exponent in LLA. The large correction indicates a poor convergence of the perturbative series. Initially this result led to serious doubts about the BFKL approach in NLLA. These doubts have been considerably weakened after the problem was subsequently studied in more detail. The large corrections were found to originate from collinear divergences due to the emission of real gluons that are close to each other in rapidity. Several methods have been proposed to circumvent this problem, among them the application of a Brodsky–Lepage–Mackenzie (BLM) scale setting procedure [26], a method to veto the emission of gluon pairs close in rapidity in a Monte Carlo implementation of the BFKL equation [27], and a renormalization group improvement of the BFKL equation resumming additional large logarithms of the transverse momentum [28]. The result of the latter procedure is shown in figure 7. The figure shows the dependence of the BFKL exponent ω_{BFKL} on the strong coupling α_s. The short-dashed line shows the exponent in LLA, the dotted line is the exponent according to NLLA without resummation as given in (3). The solid line is the result obtained after applying the renormalization group improvement. All of the methods mentioned above lead to stable results for the BFKL exponent, although the precise values differ slightly for the different methods. With those improvements the BFKL equation in NLLA is now widely considered a reasonable approximation scheme. For typical values of α_s around 0.2 one now obtains a typical exponent for the energy dependence of about 0.2 to 0.3, to be compared with the LLA value of 0.5.

An obvious question arising here is whether the BFKL Pomeron in NLLA can describe the data for the total hadronic cross section in virtual photon collisions discussed above. At present the answer to that question is unknown. At high energies the cross
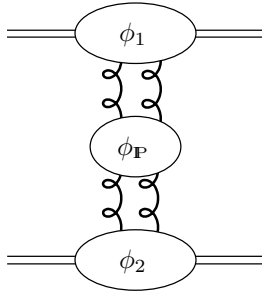
Figure 8: Factorization of the perturbative Pomeron amplitude in the high energy limit

section for a given process can be factorized in the form shown in figure 8. Obviously, the cross section involves not only the Pomeron amplitude ϕ_ω, but also the impact factors $\phi_{1,2}$ which describe the coupling of the Pomeron to the external particles. The Pomeron amplitude is known in NLLA, but a consistent calculation requires also the impact factors in that approximation. At the time of this conference they are not yet fully known. So far, the real and virtual parts of these impact factors have been calculated separately, but there remain a number of phase space integrals to be done, see [29, 30] and references therein.

We should emphasize that in spite of all the uncertainties and problems mentioned above it is a rather firm prediction of perturbative QCD that the cross section of two small color dipoles eventually rises with the energy. Practically, however, there are many caveats in applying the LLA or NLLA to a specific process at a given energy. It is therefore very difficult to isolate the perturbative Pomeron experimentally. Personally, I would guess that the total hadronic cross section in virtual photon collisions at a future Linear Collider will offer the best chances, in particular due to the larger energy and luminosity. Notwithstanding the problems of observing the perturbative Pomeron the resummation of logarithms of the energy is extremely valuable for studying perturbative QCD, in particular because it gives us information about the small-x anomalous dimension of the gluon.

3 The Odderon

The Odderon is the $C = -1$ partner of the Pomeron. It is defined as the leading contribution to the odd–under–crossing amplitude at high energies with an intercept $\alpha_{\mathbb{O}}$ close to one. Due to its negative charge parity the Odderon gives a contribution to the difference of particle–particle and particle–antiparticle cross sections. The Odderon was introduced in the framework of Regge theory almost thirty years ago in [31], but was for a long time considered a heretic and doubtful concept. Initially, one of the reasons for this was that the widely known Pomeranchuk theorem [32] states that the cross sections for particle–particle and particle–antiparticle scattering become equal at high energies. For the specific example of pp and $p\bar{p}$ scattering this means

$$\Delta\sigma = \sigma_T^{\bar{p}p} - \sigma_T^{pp} \xrightarrow[s\to\infty]{} 0 \, . \tag{4}$$

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However, the proof of this theorem *assumes* that the odd–under crossing amplitude vanishes. Without that assumption, one can show that

$$\frac{\sigma_T^{\bar{p}p}}{\sigma_T^{pp}} \xrightarrow[s\to\infty]{} 1\,, \tag{5}$$

which does not contradict the existence of the Odderon as can be seen in the following toy example for a possible behavior of the two cross sections:

$$\sigma_T^{pp} = A\log^2 s + B\log s + C \tag{6}$$
$$\sigma_T^{\bar{p}p} = A\log^2 s + B'\log s + C'\,. \tag{7}$$

Clearly, if $B \neq B'$ the general Pomeranchuk theorem (5) is satisfied, but the original Pomeranchuk theorem (4) is violated, and instead in this particular example one even has $|\Delta\sigma| \to \infty$ for $s \to \infty$.

The Odderon received more attention after it had been observed that in QCD it can be build as a state of three gluons in a symmetric color state. A Pomeron in the simplest picture consists of a two–gluon exchange, and at least the nonperturbative version of the Pomeron clearly exists and has been observed in many scattering processes. There is a priori no reason why an exchange of three gluons should not exist. This simple observation strongly suggests that an Odderon of some kind, be it perturbative or nonperturbative, should occur in high energy scattering. In the following I will briefly mention some basic facts about the perturbative Odderon and then concentrate on phenomenological issues. For a more detailed review on the Odderon see [33].

In perturbative QCD the Odderon is described by the Bartels–Kwieciński–Praszałowicz (BKP) equation [34, 35]. In analogy to the BFKL equation it resums the leading logarithms of the energy $\sqrt{s}$ for a state of three (reggeized) gluons in the t-channel. The corresponding diagrams again have a ladder structure with pairwise interactions of the three gluons as is illustrated in figure 9. Remarkably, the BKP equation has a
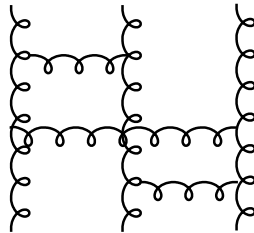

Figure 9: Typical ladder diagram contributing to the perturbative Odderon

hidden conserved charge [36, 37] and is therefore a completely integrable system. It in fact turns out that it is equivalent to the XXX Heisenberg model consisting of three sites with noncompact $\mathrm{SL}(2,\mathbb{C})$ spin $s = 0$ [38, 39].

In the past years explicit solutions of the BKP equation have been found, and it is likely that by now all solutions of the BKP equation are known. The first type of solution was found in [40] and is called the Janik–Wosiek (JW) solution. Its intercept is was found to be

$$\alpha_{\mathbf{O}} = 1 - 0.24717\,\frac{\alpha_s N_c}{\pi}\,, \tag{8}$$

and for a typical value $\alpha_s \simeq 0.2$ this yields $\alpha_{\mathbb{O}} \simeq 0.96$. Consequently one obtains an almost flat energy dependence of $s^{\alpha_{\mathbb{O}}-1}$ for the Odderon exchange in the case of the JW solution. Another type of solution of the BKP equation, the Bartels–Lipatov–Vacca (BLV) solution, was found in [41]. This solution has been constructed explicitly in terms of eigenfunctions of the BFKL equation. The intercept of the BLV Odderon solution is exactly $\alpha_{\mathbb{O}} = 1$. The exchange of the BLV Odderon should hence persist to very high energies. Naively one would conclude here that the leading contribution to the Odderon comes from the solution with the highest intercept, i. e. the BLV solution. However, the situation is more complicated. The complication comes due to the fact that the two types of solutions exhibit a very different behavior concerning the coupling to external particles. The JW solution for example does not couple (at least in leading logarithmic order) to the phenomenologically interesting photon-η_c impact factor. The BLV solution on the other hand does couple to this impact factor. As a consequence only the BLV solution contributes to the quasidiffractive process $\gamma^* \gamma^* \to \eta_c \eta_c$, for instance. In other processes, for example in proton–proton scattering at large t both solutions are expected to contribute. In such processes the relative importance of the two solutions is mainly determined by the coupling to the external particles because their intercepts are almost equal. There are strong indications that in most processes of phenomenological interest the BLV solution gives the leading contribution, and sometimes the only one.

We have seen that at least in perturbative QCD the occurrence of the Odderon is very natural. There is no obvious reason why processes involving Odderon exchange should have a very small cross section. This is supported by the fact that the perturbative Odderon intercept is exactly one (or only slightly below one for the JW solution). The situation is less clear when one turns to processes in which perturbation theory is not applicable. Only very little is known about the soft or nonperturbative Odderon. But also at low momentum scales there is no obvious reason for the absence of the Odderon. In the contrary, due to the larger value of the coupling constant at small momenta a three–gluon (Odderon) exchange should be even less suppressed with respect to the two–gluon (Pomeron) exchange. Our picture of high energy scattering based on gluon exchange, be it perturbative or nonperturbative, hence strongly suggests that Odderon exchange should exist and should lead to sizable cross sections. Reality seems to be different.

So far the only evidence for the Odderon has been found in the difference of the differential cross sections for elastic pp and $p\bar{p}$ scattering

$$\frac{d\sigma_{el}^{\bar{p}p}}{dt} - \frac{d\sigma_{el}^{pp}}{dt} \tag{9}$$

in the dip region around $t \simeq -1.3\,\mathrm{GeV}^2$. Figure 10 shows the corresponding data taken at $\sqrt{s} = 53\,\mathrm{GeV}$ at the CERN ISR [42]. The pp data show a dip whereas the $p\bar{p}$ data only flatten off at the same momentum transfer. Such a difference is a typical sign of an exchange carrying negative charge parity. It can be shown that pure reggeon exchange is not sufficient to produce the observed difference, and hence the data indicate an Odderon contribution. But there are two caveats here. First, the statistics of the data shown in figure 10 is rather low. The $p\bar{p}$ data have in fact been taken only during the last week of running of the ISR. Unfortunately, there is no other energy at which we have data for both pp and $p\bar{p}$ elastic scattering. Comparing the two at largely different energies necessarily requires theoretical models for the description
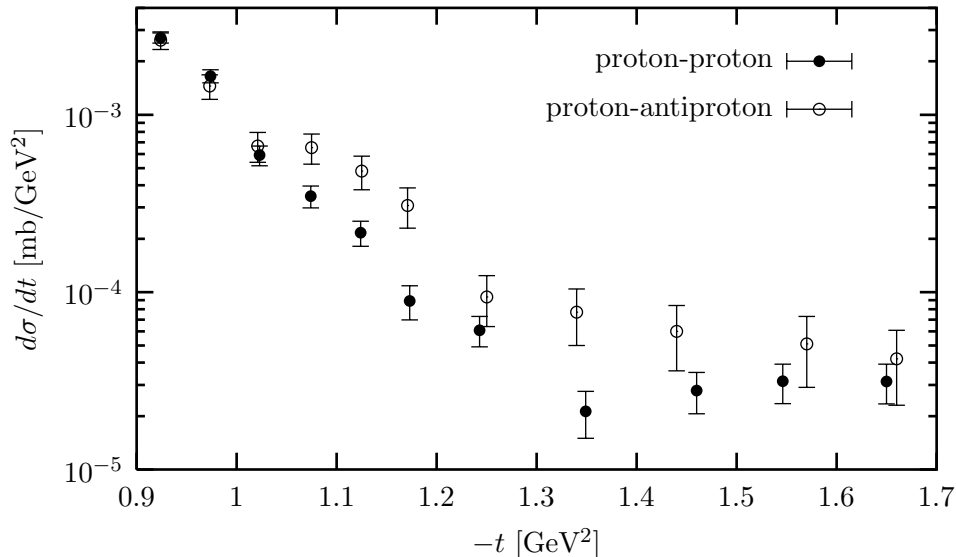
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Figure 10: Differential cross section for elastic pp and $p\bar{p}$ scattering in the dip region for $\sqrt{s} = 53\,\mathrm{GeV}$; data from [42], figure from [33]

of the data which are mostly in the soft (nonperturbative) region. This brings us to the second caveat. The available models for the elastic scattering data involve a large number of exchanges in addition to the Odderon, like Pomeron, double Pomeron, reggeon etc. Accordingly, these models have typically around 20 to 40 parameters. With the available data these simply cannot be determined precisely enough to cleanly identify the Odderon contribution. We hope that elastic scattering data from RHIC will improve this situation in the future. It is worth noting though that almost all fits require an Odderon contribution of some kind in order to describe the available data.

Although the models necessary for the description of the differential cross sections of elastic pp and $p\bar{p}$ scattering do not allow one to extract the Odderon contribution with sufficient precision it is still possible to gain interesting information on the Odderon from these data. In [43] for example the coupling of the Odderon to the proton was investigated and found to depend strongly on the internal structure of the proton. As a framework the Regge description of the elastic scattering data due to Donnachie and Landshoff (DL) [44] is used. Then the Odderon contribution in that fit is replaced by a perturbative three–gluon exchange, hoping that in the dip region the momentum transfer is still large enough to allow for a perturbative picture, at least in some reasonable approximation. One can then use different models for the coupling of the Odderon to the proton and try to determine their parameters. It turns out that in this way one can, within a given model, constrain the parameters very strongly. Therefore it is likely that the qualitative results are rather independent of the parametrization chosen as a framework.

In [43] this has been done for two impact factors that had been proposed in [45] and [46], respectively, and also for a simple geometric model for the transverse structure of the proton. The two impact factors contain as a main parameter the value of the
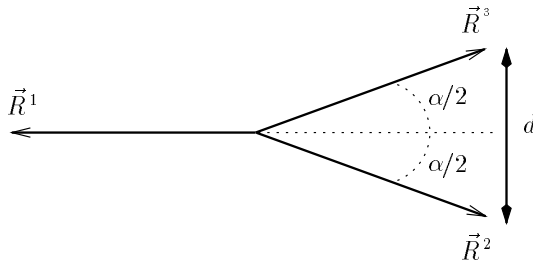
Figure 11: Definition of the angle α characterizing the proton configuration, figure from [43]

coupling α_s. The geometric model assumes a distribution of the three valence quarks in the proton of the form shown in figure 11. Here the main parameter is the size d of a pair of quarks, whereas the radius of the configuration is essentially fixed by the electromagnetic radius of the proton. In this model a possible quark–diquark structure of the proton would simply correspond to a small value of d (or a small angle α). Figure 12 shows that the parameters of all models for the Odderon–proton coupling can be chosen in such a way that the description of the data is as good as with the original DL fit (and a better description can hardly be expected within that framework). One finds that the optimal values for the coupling constant in the impact factors should be chosen relatively small, in the range $\alpha_s \simeq 0.3 - 0.5$. In the geometric model of the proton one finds that a good description of the data requires a relatively small diquark cluster in the proton, with a size of less than 0.35 fm.

It is worth noting here that originally larger values had been proposed for α_s in the impact factors in [45] and [46]. In the case of [45] even $\alpha_s = 1$ was suggested, and that value has also been used in a number of predictions for other processes involving momentum scales similar to the value of $\sqrt{-t}$ in the dip region. Since the cross section for these processes depends on α_s very strongly, like α_s^6, the original choice for its value is likely to overestimate the cross section considerably. Independent of those two models for the Odderon–proton coupling we make a seemingly trivial but very important observation here. The cross section for processes with an Odderon exchange depend very strongly on α_s, namely with a high power. Therefore even a small uncertainty in the choice of the correct value (or momentum scale) for α_s leads to the largest uncertainty of the prediction of the cross section. This significantly contributes to the difficulties in finding the Odderon.

Also the so–called ρ-parameter

$$\rho(s) = \frac{\mathrm{Re}\,A(s,t=0)}{\mathrm{Im}\,A(s,t=0)} \tag{10}$$

was for some time considered to be an observable which would be suitable for finding the Odderon. A sign of the Odderon could be a nonvanishing difference of the ρ-parameters for pp and $p\bar{p}$ scattering,

$$\Delta\rho(s) = \rho^{p\bar{p}}(s) - \rho^{pp}(s) \tag{11}$$

at high energies. Unfortunately a precise determination of the ρ-parameter is extremely
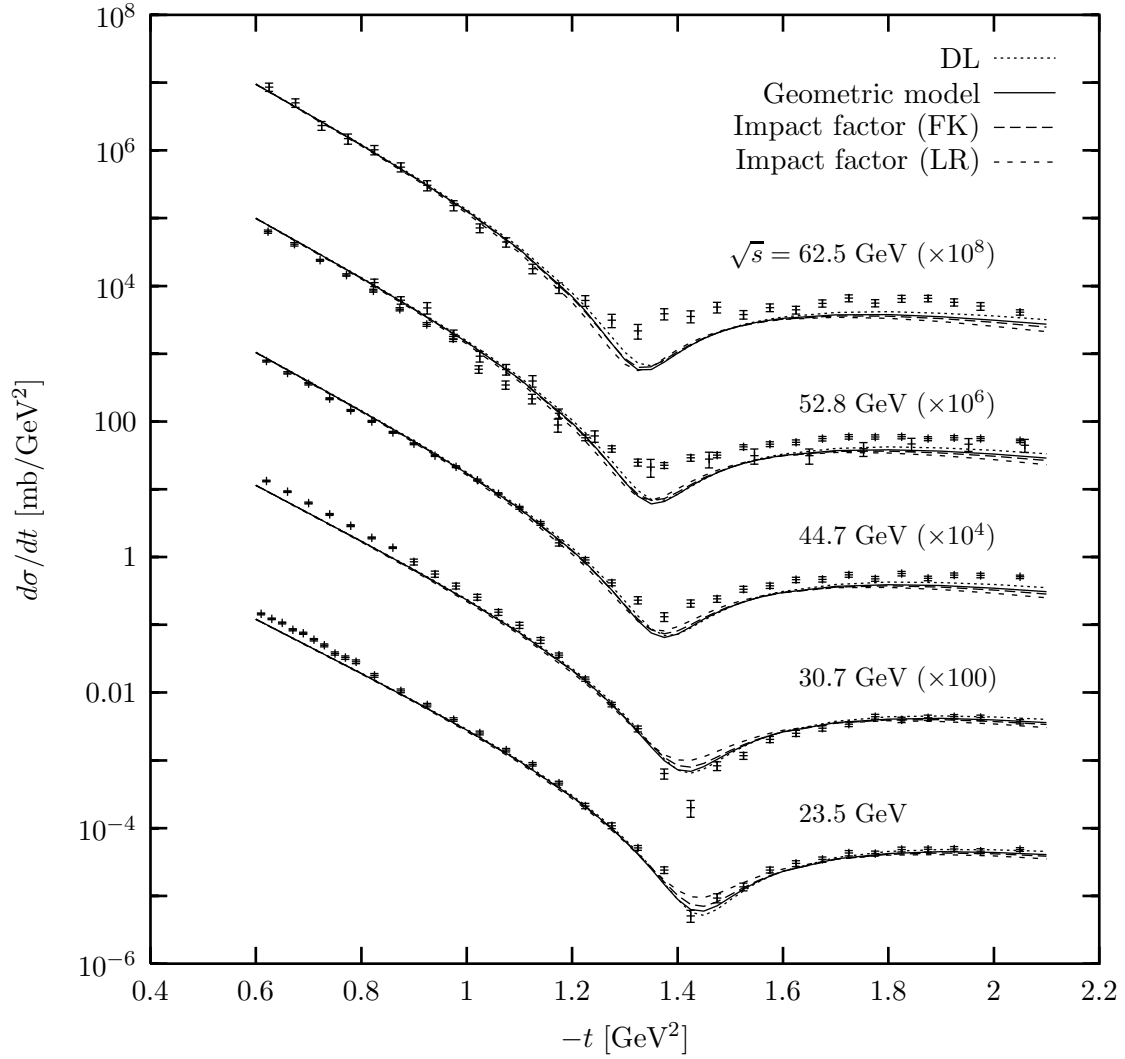
Figure 12: Differential cross section for elastic *pp* scattering calculated using different couplings of the Odderon to the proton: the original Donnachie–Landshoff fit (dotted), the geometrical model for the proton (solid), and the impact factors of [45] (FK, long–dashed) and [46] (LR, short–dashed), figure from [43]
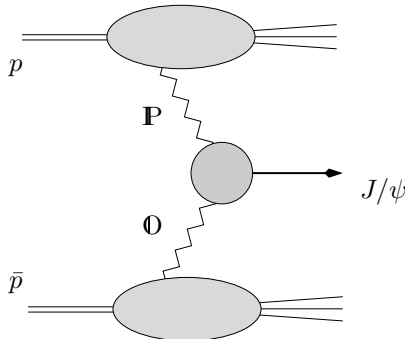
15

Figure 13: Pomeron–Odderon fusion mechanism for double–diffractive J/ψ production in $p\bar{p}$ scattering

difficult experimentally. According to the latest measurement by the UA4/2 collaboration at the CERN SPS the value of $\Delta\rho$ is compatible with zero [47]. However, it should be emphasized that even $\Delta\rho = 0$ would not exclude an Odderon, but would only rule out specific models for the soft Odderon. In total, the situation of the ρ-parameter is not really conclusive.

In the processes we have discussed so far the Odderon gives only one among many contributions to the cross section and is hence rather difficult to identify, especially because the other contributions are of nonperturbative nature and often only poorly known. In the past few years there has been a change of direction in the search for the Odderon. In order to avoid that main problem one now looks for exclusive processes in which the Odderon is the only contribution to the scattering amplitude, with the possible exception of a reggeon that is rather well understood. These processes have in general a much smaller cross section. But they have the advantage that already their observation would establish the existence of the Odderon.

A process of this kind are for example the double diffractive production of vector mesons in pp collisions, $p\,p \to p\,p\,M_V$, see figure 13, in which the vector meson would be produced via a Pomeron–Odderon fusion mechanism. This process has been considered for example in [48]. Another interesting possibility is the diffractive production of pseudoscalar or tensor mesons in ep collisions due to $\gamma^{(*)}p \to p\,M_{PS/T}$, as illustrated in figure 14. In both processes the quantum numbers of the produced mesons require an Odderon exchange.

Let us consider in some more detail the diffractive production of pseudoscalar mesons. The largest cross sections are clearly to be expected for the photoproduction of light mesons. Here both the real photon as well as the light mesons require a completely nonperturbative treatment. This process has therefore been studied in the framework of Regge theory in [49], and using more sophisticated methods in [50]. The latter approach makes use of an implementation [51, 52, 53, 54] of the stochastic vacuum model (SVM) of [55, 56, 57] in the framework of a nonperturbative framework for the description of high energy scattering [58]. That approach has proven to be very successful in the case of scattering processes mediated by Pomeron exchange. One should again expect a suppression of the Odderon–proton coupling due to a potential
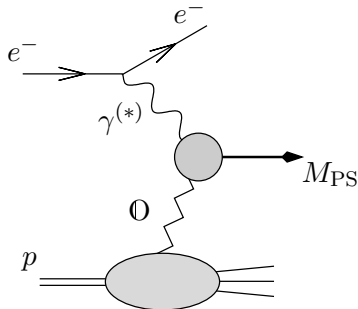
Figure 14: Diffractive production of a pseudoscalar meson in ep scattering

diquark clustering within the proton. This can be avoided if one considers only the case in which the proton breaks up. The resulting estimate for pion production in the process $\gamma p \to \pi^0 N^*$ at HERA is $\sigma \simeq 200\,\mathrm{nb}$. A similar estimate for the production of f_2 tensor mesons in $\gamma p \to f_2 X$ at HERA is $\sigma \simeq 21\,\mathrm{nb}$ [59]. Given these numbers there seemed to be a realistic chance of observing the Odderon in these processes at HERA. Figure 15 shows the rather disappointing result of a measurement of diffractive pion production [60]. The data points are far below the expectations of a Monte Carlo based on the results of [50], and are well compatible with the expected background. The calculations of [50] rely on nonperturbative techniques and naturally have a large uncertainty. Nevertheless, the rather dramatic failure of the prediction in this case is quite puzzling and is not yet understood. Possible reasons might be an extremely low intercept of the soft Odderon, a suppression of the Odderon–proton coupling due to some reason possibly involving some assumptions made in the MSV, or finally a suppression of the Odderon–pion coupling possibly related to the special role of the pion as a Goldstone boson. The latter possibility can in principle be tested by measuring the corresponding cross sections for other pseudoscalar or tensor mesons. Most probably we are missing an important insight here, and further study is urgently needed.

Less uncertainties are involved if one considers the diffractive production of heavy pseudoscalar mesons like the η_c. The mass of the charm quark provides a large scale and hence one can approach this process using perturbation theory [61, 62, 63]. In the first two references a simple three–gluon exchange is used for the Odderon without resummation, whereas in [63] the resummed BLV Odderon solution is used. The expected cross sections for the photoproduction of η_c range up to about $50\,\mathrm{pb}$, and even these values are obtained only with a very optimistic choice of α_s in the impact factors coupling the Odderon to the proton, see the discussion above. More realistically one should expect a factor 30 less. But even the optimistic estimate would be much to small for a realistic chance of observing this process at HERA.

The cross sections for the diffractive processes discusses above contain the square of the Odderon amplitude and thus have an enhanced sensitivity to the uncertainties of this amplitude like the coupling of the Odderon to external particles etc. Interestingly it is also possible to find observables which are only linear in the Odderon amplitude. If one considers final states which can be produced both via Pomeron and Odderon exchange there can be Pomeron–Odderon interference effects [64] which typically occur
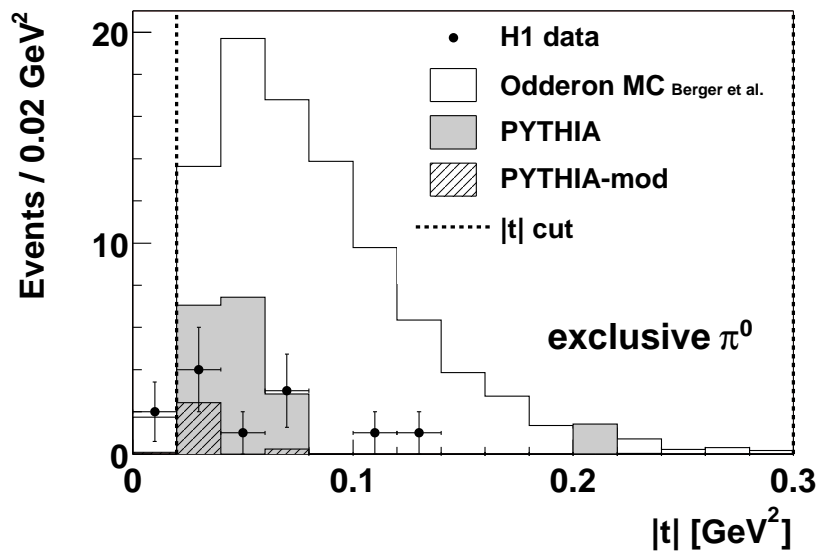
Figure 15: Measured t-distribution for Odderon candidate events with $M_{\gamma\gamma} < 335\,\text{MeV}$; figure from [60]

in suitably constructed asymmetries. On the parton level these asymmetries usually vanish, but in the process of hadronization there occur additional Breit–Wigner phases which can give rise to a sizable effect in such asymmetries. The violation of quark–hadron duality is therefore crucial for these observables. Probably the most favorable observable of this kind is the charge asymmetry in diffractive $\pi^+\pi^-$ production in ep collisions at HERA. The charge asymmetry is defined as

$$\mathcal{A}(Q^2, t, m^2_{\pi^+\pi^-}) = \frac{\int \cos\theta \, d\sigma(s, Q^2, t, m^2_{\pi^+\pi^-}, \theta)}{\int d\sigma(s, Q^2, t, m^2_{\pi^+\pi^-}, \theta)} \,, \tag{12}$$

where θ is the polar angle of the π^+ in the dipion rest frame. A pion pair can be produced both in a C-odd and in a C-even state, the former via Pomeron exchange and the latter via Odderon exchange. The charge asymmetry is constructed such that it is given by the interference term of the Pomeron and the Odderon amplitude. One expects the asymmetry to be of the order of about $10 - 20\,\%$ for both photoproduction [65, 66] and electroproduction [67, 68]. A measurement of the charge asymmetry would clearly be a very important step in the search for the Odderon.

4 Summary

The perturbative Pomeron and the corresponding rise of cross sections are firm predictions of QCD. But as in the case of any other approximation in physics one has to identify the appropriate range of applicability of the BFKL Pomeron. Contrary to earlier expectations it turns out that the perturbative Pomeron applies only in a limited number of experimentally testable scattering processes, and even there only in suitable

kinematic situations. In these, there are in fact some indications for a behavior compatible with the perturbative Pomeron. The nonobservation of the perturbative Pomeron in other processes, on the other hand, should actually not cause any excitement.

If our understanding of high energy scattering based on gluon exchanges is correct there should or even must be an Odderon of some kind. This holds in particular in situations in which perturbation theory is applicable. In situations involving lower momentum scales, however, it is conceivable that Odderon exchange is suppressed in some form, for example due to a very low intercept of the nonperturbative Odderon.

This conference took place in the building of the Heidelberger Akademie der Wissenschaften. Besides the warm hospitality of the staff of the Akademie we also enjoyed an impressive view on the Heidelberg castle. The poetic touch of this place can hardly be matched by any scientific statement. Nevertheless, I would like conclude with another version of my summary of the status of the Odderon that is slightly more poetic than the one given above:[2]

Das Odderon, das Odderon
Man hat gehört davon.
Es beliebt sich sehr zu zieren,
will man es detektieren.
Nebulös sind alle Zeichen,
ein Knick in Sigma, der muß reichen.
Doch daß die Daten hier sich beugen,
vermag so manchen nicht zu überzeugen.
Ein Eta, diffraktiv erschienen,
würde allerliebst uns dienen.
Jedoch, die Suche bleibt sehr schlicht,
man findets einfach nicht.
Und wenn wir noch so hoffen,
am Ende ist wie immer alles offen.

Acknowledgements

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[2]I have chosen the German (that is the local) language here since my ability to make rhymes is somewhat limited in other languages.

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